Final Exam 7/202X
Twenty components were used in a high-stress environment. After 15 hours of use, seven of them failed at the following times (in hours):
2.1, 8.3, 10.9, 15.2, 16.3, 20.5, 23.8
The remaining 13 components were still functioning.
Calculate the following parameters:

1. Mean Time to Failure (MTTF)
2. The Failure Rate
3. The time at which the reliability is 95%
4. The Reliability after 50 hours of use

SOLUTION:

Based on the information shown on the whiteboard in the image, the calculations differ slightly from the previously provided values. Here's a comparison and explanation of the calculations seen in the image:

1. **MTTF (Mean Time to Failure):**
    - From the image

$$MTTF = \frac{\text{Sum of failure times}}{\text{Number of failures}} = \frac{422.1}{7} = 60.3 \text{ hours}$$

- My calculation used the average of failure times directly, which gave around 13.87 hours. It seems the board calculation accounts for all failed and unfailed times differently.

2. Failure Rate (λ):

- From the image:

$$\lambda = \frac{1}{MTTF} = \frac{1}{60.3} = 0.0166 \text{ failures per hour}$$

- My calculated rate was 0.00937 per hour due to a different MTTF assumption.

3. Time at which reliability is 95%:

- From the image:

$$0.95 = e^{-0.0166t} \implies t = 3.09 \text{ hours}$$

- The discrepancy with my value comes from the difference in the failure rate.

4. Reliability after 50 hours:

- From the image:

$$R(50) = e^{-0.0166 \times 50} = 43.6\%$$

- My calculation showed 62.6%, again due to differences in the failure rate used.

Probability

When estimating the probability of a complex system, like an Aircraft Carrier or an Oil Rig, it is impractical to test these systems to destruction. Therefore, the reliability of these systems is calculated by estimating the reliability of individual components.

Rule #1: If A and B are two events of interest, and P_A and P_B are their respective probabilities of occurring, then if A and B are independent, their combined probability is:

$$P(A\&B) = P_A \times P_B$$

Rule #2: If A and B are mutually exclusive, then:

$$P(A \text{ or } B) = P_A + P_B$$

Rule #3: If $A_1, A_2, A_3, \ldots, A_n$ are mutually exclusive events and they describe all possible outcomes in a particular situation, then:

$$P_1 + P_2 + P_3 + \ldots + P_n = 1$$

Rule #4: If there are only two possible outcomes (say, success and failure), the probability of success can be expressed as:

$$P(\text{success}) = 1 - P(\text{failure})$$

Note:

$$P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total possible number of outcomes}}$$

So, the probability of event A occurring is always between 0 and 1:

$$0 \leq P(A) \leq 1$$

Sample Space: A sample space is defined as a set of all possible outcomes that can occur.

TREE DIAGRAM OF THREE COINs

Tree Diagram Structure:

1. **First Level**:
   - **H**
   - **T**
2. **Second Level** (from each first level outcome):
   - **H → H**
   - **H → T**
   - **T → H**
   - **T → T**
3. **Third Level** (from each second level outcome):
   - **H → H → H**

- **H → H → T**
- **H → T → H**
- **H → T → T**
- **T → H → H**
- **T → H → T**
- **T → T → H**
- **T → T → T**

Diagram Representation:

Outcomes:

1. HHH
2. HHT
3. HTH
4. HTT
5. THH
6. THT
7. TTH
8. TTT

Indefinite integral

To compute an indefinite integral, we need a specific function to integrate. The indefinite integral, also known as the antiderivative, is the process of finding a function $F(x)$ whose derivative is the given function $f(x)$.

The general form is:

$$\int f(x)\,dx = F(x) + C$$

where:

- $f(x)$ is the function to integrate,
- $F(x)$ is the antiderivative of $f(x)$,
- C is the constant of integration.

Example of Basic Indefinite Integrals:

1. $\int x^n\,dx$:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (\text{where } n \neq -1)$$

2. $\int e^x\,dx$:

$$\int e^x\,dx = e^x + C$$

3. $\int \sin(x)\,dx$:

$$\int \sin(x)\,dx = -\cos(x) + C$$

4. $\int \cos(x)\,dx$:

$$\int \cos(x)\,dx = \sin(x) + C$$

5. $\int \frac{1}{x}\,dx$:

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Q A six sided die is tossed
(a) what is the probability of getting a '2'
(b) what is the " " " '3' or '5'
(c) what is the " " " " number that is atmost 4
(d) What is the " " " " number that is greater than 3
(e) What is the " " " " number that is less than or equal to 5

(a) What is the probability of getting a 2?

A six-sided die has the numbers 1, 2, 3, 4, 5, and 6. The probability of rolling a specific number is:

$$P(\text{getting a } 2) = \frac{1}{6}$$

(b) What is the probability of getting a 3 or 5?

For rolling a 3 or a 5, there are two favorable outcomes: 3 and 5. Therefore, the probability is:

$$P(\text{getting a } 3 \text{ or } 5) = \frac{2}{6} = \frac{1}{3}$$

(c) What is the probability of getting a number that is at most 4?

The numbers that are "at most 4" are 1, 2, 3, and 4. Therefore, there are 4 favorable outcomes. The probability is:

$$P(\text{at most } 4) = \frac{4}{6} = \frac{2}{3}$$

(d) What is the probability of getting a number that is greater than 3?

The numbers greater than 3 are 4, 5, and 6. Therefore, there are 3 favorable outcomes. The probability is:

$$P(\text{greater than } 3) = \frac{3}{6} = \frac{1}{2}$$

(e) What is the probability of getting a number that is less than or equal to 5?

The numbers less than or equal to 5 are 1, 2, 3, 4, and 5. Therefore, there are 5 favorable outcomes. The probability is:

$$P(\text{less than or equal to } 5) = \frac{5}{6}$$

Q

A jar contains 7 red marbles, 6 green marbles,
5 blue marbles and 2 yellow marbles.

(a) What is the probability of selecting a green marble.

(b) " " " " " " " blue "

(c) Green or Yellow

(d) Red then blue with replacement

(e) Red then blue without replacement.

(f) Red & blue without replacement

(g) What is the prob of selecting Red then green marble or getting a blue marble and then Yellow marble with replacement.

We have:

- 7 red marbles
- 6 green marbles
- 5 blue marbles
- 2 yellow marbles

Total marbles:

$$7 + 6 + 5 + 2 = 20$$

(a) What is the probability of selecting a green marble?

The probability is the number of green marbles divided by the total number of marbles:

$$P(\text{green}) = \frac{6}{20} = \frac{3}{10}$$

(b) What is the probability of selecting a blue marble?

The probability is the number of blue marbles divided by the total number of marbles:

$$P(\text{blue}) = \frac{5}{20} = \frac{1}{4}$$

(c) What is the probability of selecting a green or yellow marble?

The probability of green or yellow is the sum of the probabilities of selecting a green marble and a yellow marble:

$$P(\text{green or yellow}) = P(\text{green}) + P(\text{yellow}) = \frac{6}{20} + \frac{2}{20} = \frac{8}{20} = \frac{2}{5}$$

(d) What is the probability of selecting a red marble, then a blue marble (with replacement)?

Since the marbles are replaced after each selection, the probabilities remain the same for both selections. The probability of selecting a red marble followed by a blue marble is:

$$P(\text{red then blue}) = P(\text{red}) \times P(\text{blue}) = \frac{7}{20} \times \frac{5}{20} = \frac{35}{400} = \frac{7}{80}$$

(e) What is the probability of selecting a red marble, then a blue marble (without replacement)?

Since there is no replacement, the total number of marbles decreases after the first draw. The probability of selecting a red marble and then a blue marble is:

$$P(\text{red then blue without replacement}) = \frac{7}{20} \times \frac{5}{19} = \frac{35}{380} = \frac{7}{76}$$

(f) What is the probability of selecting a red marble and a blue marble (without replacement)?

For this, we need to account for two scenarios: drawing red first, then blue, or drawing blue first, then red.

- Probability of red first, then blue:

$$P(\text{red then blue}) = \frac{7}{20} \times \frac{5}{19} = \frac{7}{76}$$

- Probability of blue first, then red:

$$P(\text{blue then red}) = \frac{5}{20} \times \frac{7}{19} = \frac{7}{76}$$

Thus, the total probability of selecting a red and a blue marble (in either order) is:

$$P(\text{red and blue}) = \frac{7}{76} + \frac{7}{76} = \frac{14}{76} = \frac{7}{38}$$

(g) What is the probability of selecting a red marble, then a green marble, and then a yellow marble (with replacement)?

Since each marble is replaced, the probabilities remain the same for each selection. The probability of selecting a red marble, followed by a green marble, followed by a yellow marble is:

$$P(\text{red, then green, then yellow}) = P(\text{red}) \times P(\text{green}) \times P(\text{yellow}) = \frac{7}{20} \times \frac{6}{20} \times \frac{2}{20} = \frac{84}{8000} = \frac{21}{2000}$$

1. **Note on Dice and Coin Tossing:**

- Dice and coin tosses are considered **independent events**.
- These events do not "remember" previous outcomes, meaning past results do not influence future ones.
- They are **unbiased** and have equally probable outcomes, which occur randomly.

2. **Addition Rule:**

- For two events A and B, the probability of either A or B happening is given by:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- This formula ensures that we do not double-count the overlap (the event where both A and B happen together).
- **Mutually Exclusive Events:**
  - If A and B are mutually exclusive (they cannot occur at the same time), then:

$$P(A \cup B) = P(A) + P(B)$$

  - This is because $P(A \cap B) = 0$ for mutually exclusive events.

3. **Multiplication Rule:**

- **Conditional Probability:** If two events A and B are not independent, the probability of A given that B has already occurred is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- The multiplication rule for finding the probability of both A and B happening (dependent events) is:

$$P(A \cap B) = P(B) \times P(A|B)$$

- For independent events, this simplifies to:

$$P(A \cap B) = P(A) \times P(B)$$

4. **Three or More Events (Addition Rule):**

- The addition rule can be extended to three or more events. For events A, B, and C, the probability of any of these events happening is:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

- This formula takes into account all overlaps (intersections) between the events.

These principles help to calculate probabilities in various scenarios, whether events are independent or dependent, or if we are dealing with multiple events simultaneously. Let me know if you need further explanations or if you'd like to go deeper into specific problems!

Given:

- 8 red marbles
- 7 blue marbles
- 6 green marbles
- 4 yellow marbles

$$8+7+6+4=25$$

(a) What is the probability of selecting a red marble?

The probability of selecting a red marble is the number of red marbles divided by the total number of marbles:

$$P(\text{red})=\frac{8}{25}$$

(b) What is the probability of selecting a blue marble on the first try and then a green marble on the second try (with replacement)?

Since the marbles are replaced after each draw, the probability for each event is independent, and the total number of marbles remains the same for both draws. The probability is:

$$P(\text{blue then green})=P(\text{blue})\times P(\text{green})=\frac{7}{25}\times\frac{6}{25}=\frac{42}{625}$$

(c) What is the probability of selecting a yellow marble on the first try and then a red marble on the second try (without replacement)?

Since there is no replacement, the total number of marbles decreases after the first draw. The probability is:

$$P(\text{yellow then red})=P(\text{yellow})\times P(\text{red without replacement})=\frac{4}{25}\times\frac{8}{24}=\frac{32}{600}=\frac{4}{75}$$

(d) What is the probability of selecting two blue marbles with replacement?

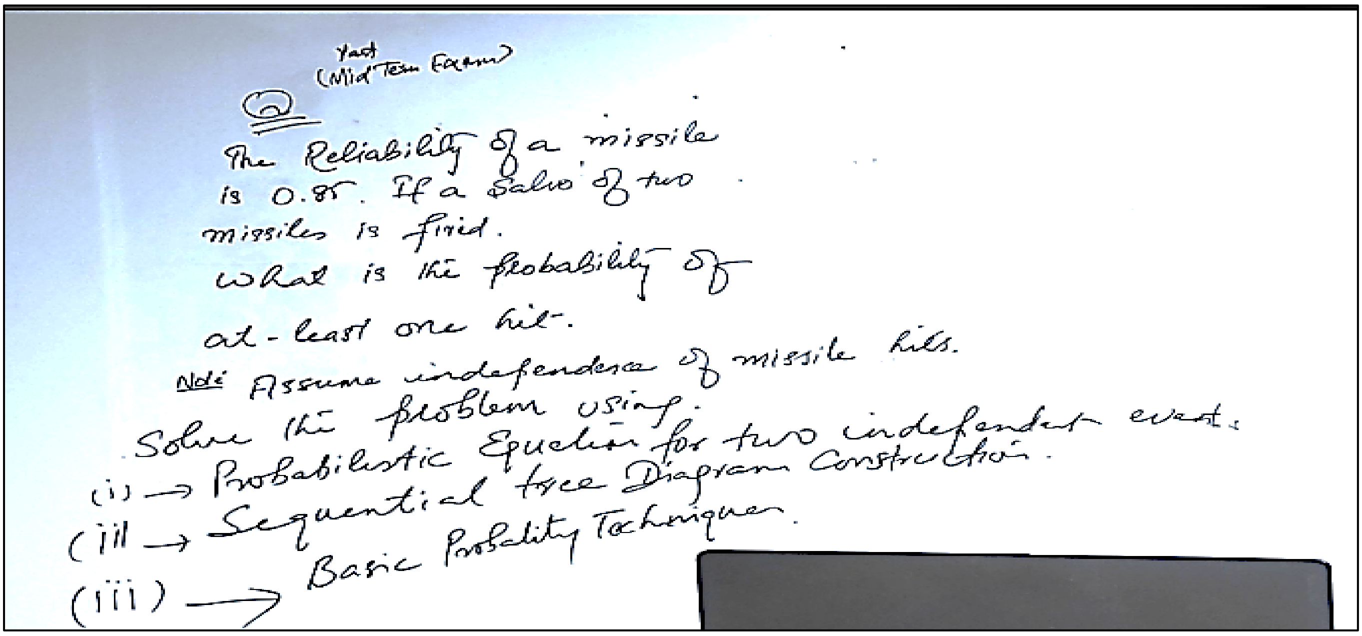
With replacement, the probability of selecting a blue marble each time is independent of the previous draw:

$$P(\text{two blue marbles with replacement})=P(\text{blue})\times P(\text{blue})=\frac{7}{25}\times\frac{7}{25}=\frac{49}{625}$$

(e) What is the probability of selecting two green marbles without replacement?

Without replacement, the probability of selecting a green marble changes after the first draw, as the total number of marbles decreases:

$$P(\text{two green marbles without replacement})=P(\text{green})\times P(\text{green without replacement})=\frac{6}{25}\times\frac{5}{24}=\frac{30}{600}=\frac{1}{20}$$



Step-by-step solution:

1. Identify the probability of failure:

- The reliability of each missile is $P(\text{hit}) = 0.85$, which means the probability of failure is:

$$P(\text{miss}) = 1 - P(\text{hit}) = 1 - 0.85 = 0.15$$

2. List all possible outcomes:

For two missiles, the possible outcomes are:

- **Hit, Hit** (Both hit): $P(\text{hit}) \times P(\text{hit}) = 0.85 \times 0.85 = 0.7225$
- **Hit, Miss** (First hits, second misses): $P(\text{hit}) \times P(\text{miss}) = 0.85 \times 0.15 = 0.1275$
- **Miss, Hit** (First misses, second hits): $P(\text{miss}) \times P(\text{hit}) = 0.15 \times 0.85 = 0.1275$
- **Miss, Miss** (Both miss): $P(\text{miss}) \times P(\text{miss}) = 0.15 \times 0.15 = 0.0225$

3. Find the probability of at least one hit:

The probability of at least one hit is the complement of both missiles missing. So, we can calculate:

$$P(\text{at least one hit}) = 1 - P(\text{both miss}) = 1 - 0.0225 = 0.9775$$

Thus, the probability of at least one missile hitting the target is approximately **0.9775**, or **97.75%**.

Methods Mentioned on the Board:

1. **Probabilistic Equation for Two Independent Events:**
   - Use the formula for independent events, as explained above. The formula is:
   $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
2. **Sequential Tree Diagram Construction:**
   - A tree diagram visually shows all possible outcomes and their corresponding probabilities. In this case, it would show the outcomes for both hitting, one hitting and the other missing, etc.

In probability, certain terms or hints in the question can guide you on how to approach solving it. Some of these terms include **"replacement," "without replacement," "exactly one," "at least,"** and **"at most."** I'll explain each term and provide simple examples to clarify the concepts.

1. **With Replacement:**

When an event involves **replacement**, it means that after you choose an item (e.g., a marble), it is **put back** into the original group, so the total number of items stays the same.

Example:

- Imagine you have a jar with 3 red marbles and 2 blue marbles.
- You randomly select one marble, record its color, and then **put it back** into the jar.
- If you draw again, the total number of marbles remains 5 (because you put the first marble back).

Scenario:

- **What is the probability of drawing a red marble twice (with replacement)?**

$$P(\text{red, then red}) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$$

Since you replaced the first marble, the total number of marbles stays the same for both draws.

2. **Without Replacement:**

When an event involves **without replacement**, it means that after choosing an item, it is **not put back** into the group. So, the total number of items decreases after each selection.

Example:

- You have the same jar with 3 red marbles and 2 blue marbles.
- You randomly select a marble, record its color, but **do not put it back** into the jar.
- The second time you select, there is **one fewer** marble in the jar.

Scenario:

- **What is the probability of drawing a red marble and then a blue marble (without replacement)?**

$$P(\text{red, then blue}) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$$

Since the first marble was not replaced, there are only 4 marbles left when drawing the second time.

3. **Exactly One:**

When the question asks for **exactly one** of something, it means that only one of the events of interest occurs, but not both.

Example:

- You flip a coin twice and are asked, "What is the probability of getting **exactly one heads?**"

Scenario:

- The possible outcomes for two coin flips are: HH, HT, TH, TT.
- Exactly one heads occurs in two cases: HT and TH.
- So, the probability is:

$$P(\text{exactly one heads}) = \frac{2}{4} = \frac{1}{2}$$

4. **At Least:**

When the question says **at least**, it means that the event of interest happens **that many times or more.**

Example:

- You roll a six-sided die. "What is the probability of rolling at least a 4?"

Scenario:

- The possible outcomes are 1, 2, 3, 4, 5, 6.
- "At least a 4" means 4, 5, or 6 (so 3 outcomes).
- The probability is:

$$P(\text{at least } 4) = \frac{3}{6} = \frac{1}{2}$$

5. **At Most:**

When the question says **at most**, it means the event of interest happens **that many times or less**.

Example:

- You roll a six-sided die. "What is the probability of rolling at most a 3?"

Scenario:

- The possible outcomes are 1, 2, 3, 4, 5, 6.
- "At most a 3" means 1, 2, or 3 (so 3 outcomes).
- The probability is:

$$P(\text{at most } 3) = \frac{3}{6} = \frac{1}{2}$$

6. **Complement (Not Event):**

When the question asks for the **complement** of an event, it means the probability of the event **not happening**.

Example:

- A bag contains 3 red and 2 blue marbles. "What is the probability of **not drawing** a red marble?"

Scenario:

- The probability of drawing a red marble is $\frac{3}{5}$.
- The probability of **not drawing a red marble** is:

$$P(\text{not red}) = 1 - \frac{3}{5} = \frac{2}{5}$$

7. **Independent Events:**

Events are **independent** when the outcome of one does not affect the outcome of the other.

Example:

- Flipping a coin twice. The result of the first flip does not influence the second.

Scenario:

- "What is the probability of flipping heads twice?"
- The probability of heads on each flip is $\frac{1}{2}$.
- Since the events are independent, the probability of both heads is:

$$P(\text{heads, then heads}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

8. **Dependent Events:**

Events are **dependent** when the outcome of one event **affects** the outcome of the next.

Example:

- You have a deck of cards, and you draw two cards **without replacement.**

Scenario:

- "What is the probability of drawing an Ace and then a King (without replacement)?"
- The probability of drawing an Ace on the first draw is $\frac{4}{52}$, and after drawing the Ace, there are cards left. The probability of drawing a King is $\frac{4}{51}$.
- So, the probability of both events occurring is:

$$P(\text{Ace, then King}) = \frac{4}{52} \times \frac{4}{51} = \frac{16}{2652} = \frac{4}{663}$$

Summary Table:

Term	Meaning	Example Calculation
With replacement	The total number of items stays the same after each selection	$\frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$
Without replacement	The total number of items decreases after each selection	$\frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$
Exactly one	Only one of the events happens (not both)	$P(\text{heads}) = \frac{2}{4} = \frac{1}{2}$
At least	The event happens that many times or more	$P(\text{at least 4}) = \frac{3}{6} = \frac{1}{2}$
At most	The event happens that many times or less	$P(\text{at most 3}) = \frac{3}{6} = \frac{1}{2}$
Complement	Probability of the event **not** happening	$P(\text{not red}) = 1 - \frac{3}{5} = \frac{2}{5}$
Independent	The outcome of one event does not affect the other	$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
Dependent	The outcome of one event affects the next event	$\frac{4}{52} \times \frac{4}{51} = \frac{4}{663}$

Problem Statement:

We have two missiles, and the reliability (probability of hitting the target) for each missile is 0.85. We need to find the probability that **at least one missile** will hit the target.

Sequential Tree Diagram:

The diagram shows four possible outcomes for the two missiles (A and B):

- AB: Both missiles hit the target.
- AB': The first missile hits, but the second missile misses.
- $A'B$: The first missile misses, but the second missile hits.
- $A'B'$: Both missiles miss.

The probabilities for these outcomes are as follows:

1. **AB: Both missiles hit**

$$P(AB) = 0.85 \times 0.85 = 0.7225$$

2. **AB': The first missile hits, but the second missile misses**

$$P(AB') = 0.85 \times 0.15 = 0.1275$$

3. **$A'B$: The first missile misses, but the second missile hits**

$$P(A'B) = 0.15 \times 0.85 = 0.1275$$

4. **$A'B'$: Both missiles miss**

$$P(A'B') = 0.15 \times 0.15 = 0.0225$$

Finding the Probability of At Least One Hit:

We are interested in the probability that **at least one missile hits the target**. This can be found by **subtracting** the probability that **both missiles miss** from 1:

$$P(\text{at least one hit}) = 1 - P(A'B')$$

$$P(\text{at least one hit}) = 1 - 0.0225 = 0.9775$$

So, the probability that at least one missile hits the target is **0.9775** or **97.75%**.

Final Answer:

$$P(\text{at least one hit}) = 97.75\%$$

Data Given:

- $P(A) = 0.3$ (Probability of enrolling in Algebra)
- $P(B) = 0.7$ (Probability of enrolling in Biology)
- $P(A|B) = 0.4$ (Probability of enrolling in Algebra, given that Biology is taken)

(a) Probability that Sarah will enroll in both Algebra and Biology:

$$P(A \text{ and } B) = P(A|B) \times P(B)$$

$$P(A \text{ and } B) = 0.4 \times 0.7 = 0.28$$

So, the probability that Sarah enrolls in both courses is 28%.

(b) Probability that Sarah will enroll in either Algebra or Biology:

We use the Addition Rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = 0.3 + 0.7 - 0.28 = 0.72$$

So, the probability that she enrolls in either Algebra or Biology is 72%.

(c) Are the two events independent?

Two events are independent if:

$$P(A|B) = P(A)$$

We know:

$$P(A|B) = 0.4 \quad \text{and} \quad P(A) = 0.3$$

Since $0.4 \neq 0.3$, **the events are not independent.**

(d) Are the two events mutually exclusive?

Two events are **mutually exclusive** if they cannot happen together, meaning:

$$P(A \text{ and } B) = 0$$

But here, $P(A \text{ and } B) = 0.28$, so **the events are not mutually exclusive** because both can occur together.

1. Deterministic Failure:

- These are failures that occur in a predictable manner. They follow a clear cause-and-effect pattern, where the failure is due to some specific reasons (e.g., a component overheating after exceeding a certain temperature). These failures are not random and are often based on physical laws or known failure modes.

2. Random Failures:

- These types of failures do not occur in a predictable manner and can happen at any time. They might still follow some probability distribution, meaning that the likelihood of failure at a given time can be modeled, even though the exact moment of failure is not known.
- The diagram seems to show a probability distribution curve (like a bell curve) which is used to model these random failures. It suggests that random failures may follow a certain pattern over time, and reliability can be calculated based on this distribution.

Random Variables (RV):

- **Definition**: A random variable allows us to move from an experimental outcome to a numerical form of the outcome.
- For example, rolling a die gives an outcome (say a number), and this number is a random variable.

Types of Random Variables:

1. **Discrete Random Variables (Discrete RV)**:
    - These variables take on a countable number of distinct values.
    - Example: The result from rolling a die (values can be 1, 2, 3, 4, 5, or 6).
2. **Continuous Random Variables (Continuous RV)**:
    - These variables can take any value within a range and are not countable.
    - Example: The mass of an object, which can be any value (within a range) and could include decimals like 1.23 kg, 1.25 kg, etc.

Statistical Terms:

- **Mean**: The average value of the random variable.
- **Standard Deviation (Std Dev)**: A measure of how spread out the numbers are from the mean.
- **Sample Portion**: Refers to a subset of a population used to estimate parameters like the mean.